An Empirical Analysis of Counterparty Risk in CDS Prices

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Abstract

In volatile markets, the defaults of firms tend to trigger each other. This contagion effect is especially important in CDS markets. Thus, counterparty risk should be considered in pricing Credit Default Swaps (CDS) to capture this "credit contagion". In this paper, we propose a new simulation method that estimates CDS spreads taking into account effect of counterparty risk among the CDS buyer, CDS seller and the reference entity. We simulate the default time as the first passage times when the total asset values of each party are under their own firm-specific barriers. We incorporate the correlated first passage times in pricing the CDS. We use a sample of CDS issued on financial services firms to illustrate that the counterparty risk is not currently priced into the CDS spreads, and the CDS spreads issued by most financial institutions are over-priced.

The CDS is the most popular and common type of a credit derivative. CDS is an insurance-type contract, which provides the buyer with protection against losses in the event that a bond issued by a corporation or sovereign entity defaults. In the event of default, the protection seller (CDS writer) pays a certain amount to the protection buyer (CDS holder). CDS contracts can be settled either in cash or in a physical settlement and the default payment is designed to cover the losses a typical bondholder would experience in the event of default. In exchange for this insurance-type protection, the protection buyer makes periodic payments to the protection seller until the maturity of the contract, or until the default event occurs[1].

CDS contracts are mostly used to relieve bank balance sheets from credit risk. This credit risk transference function and potential portfolio yield improvement make CDS contracts very attractive to banks, mutual funds, pension funds, insurance companies, and hedge funds. The market for CDSs has thus grown exponentially since the first CDS was introduced by JP Morgan in 1997. According to the International Swaps and Derivatives Association (ISDA), the notional amount outstanding for CDS grows from approximately $919 billion in 2001 to approximately $30.4 trillion in 2009[2]. The European Central Bank Report indicates that the share of CDS on the entire credit derivative market is around 20% as of June 2008[3].

There are three parties, i.e. protection buyer, protection seller and the reference

[1] The details of CDS construction as they relate to the theoretical pricing model are presented in Section 1.
[2] The information is retrieved from http://www.isda.org.
[3] The information is retrieved from a report (Credit default swaps and counterparty risk, 2009) by European Central Bank.

entity, involved in a CDS contract. The early default of any party leads to the immediate change of the CDS spread. The high likelihood of the default of any party is called counterparty risk. Jarrow and Yu (2001) qualified counterparty risk as a firm specific risk. For example, the failure of the customers or suppliers is a specific risk to the firm. The default of a firm is affected not only by economic common risk, but also by firm specific risks. Jorion and Zhang (2009) found that the default of counterparty directly leads to a drop in stock prices and an increase in the CDS spread. They also suggested that the strength of the counterparty effects depends on the variables of the exposure, the recovery rate, and the previous stock return correlations. Since the value of the CDS contract depends on the counterparty risk, creditworthiness of the buyer, seller and the reference security issuer becomes a very important determinant of the CDS spread. Existing research like Deniels and Jensen (2004), Skinner and Townsend (2002), Benkert (2004) and Aunon-Nerin et al. (2002) found that implied volatility, liquidity, leverage and credit rating have significant effects on CDS spreads. One major shortcoming in the existing literature is the lack of information on counterparty risk. Jarrow and Yu (2001) pointed out that the traditional structural or reduced-form models ignore the firm specific risks. Jarrow and Yu (2001) extended the traditional reduced-form models but only focus on the default risk between protection seller and the reference entity. Leung and Kwok (2005) and Yu (2007) further expanded reduced-form models by examining the inter-dependent default risk structure among three parties: the protection buyer, protection seller and the reference entity. From Leung and Kwok (2005), the default risk of the reference entity is the major determinant of CDS spread. The effect of protection buyer default risk is the smallest among three parties, followed by the effect of protection seller default risk. According to Yu (2007), an increase of default correlation between reference entity and protection buyer will increase the CDS spread due to the increased likelihood of failing to make the periodic payments by protection buyer. A higher default correlation between reference entity and protection seller leads to a lower CDS spread since protection seller in this case offers less protection on the reference entity. By Leung and Kwok (2005), if reference entity has higher default correlations with both protection seller and protection buyer, there is no change on CDS spread. However, according to Leung and Kwok (2005), this type of reduced-form model on counterparty risk is unable to capture the intermediate changes in total asset values for each counterparty prior to a credit event.

In this paper we extend the structural model to capture the counterparty risk of CDS buyer, seller and the reference entity to determine the CDS spread. The structural model can capture the significant changes in total asset values of counterparties. We also compare these spreads with the observed historical CDS spreads from the market to determine which market proxies work the best in calculating CDS spreads. We find that the counterparty risk is not priced by credit derivative markets, and thus the CDS spreads for most financial firms are overestimated. We analyze the difference between the historical and calculated model spreads and find that the discrepancy lies in the imperfect proxies of counterparty risk in pricing CDS contracts.

Section 1 provides an in-depth discussion of CDS contracts and summarizes the existing literature on the valuation of CDS contracts. Section 2 presents the details of the model that simulates the correlated first passage times as well as the resulting CDS spreads. Section 3 summarizes the data and methodology while section 4 presents the empirical results. Section 5 concludes the paper.

1. Credit Default Swaps

1.1 Definition of Credit Default Swaps

A credit default swap is a financial contract that is negotiated between two parties, a contract seller and a contract buyer, protecting against a specified bond default. A credit default swap is similar to an insurance contract. The protection seller receives a periodic fee (credit default swap premium which is paid by the protection buyer) to compensate for undertaking the credit risk of a specified reference security before the maturity of the swap. If there is a credit event occurring with the specified reference entity, the protection seller compensates the protection buyer for the corresponding financial loss. Otherwise, the CDS contract expires at the maturity date. In a credit default event, a CDS contract can be settled with either a cash settlement or a physical settlement. Under a cash settlement, the protection seller pays the difference between the notional and the recovery value of the underlying reference obligation in the event of credit default. Under a physical settlement, the protection buyer delivers the reference obligation to the protection seller and then receives the full notional amount from the protection seller.

The following are the key terms used in a typical CDS contract:

1. *Reference obligation* - A CDS contract protects the contract buyer against a bond default. The underlying bond here is called the reference obligation. A reference obligation is usually an unsubordinated corporate bond or government bond. The contract seller only compensates the contract buyer for the financial loss of a specified bond.

2. *Reference Entity* - The issuer of the reference obligation is called the reference entity. A credit event of the reference entity leads to the default of the reference obligation.

3. *Credit Event* - A list of events that trigger the financial loss of the protection buyers due to the credit risk of reference entity are called credit events. According to the International Swaps and Derivatives Association, there are several typical credit events, which are mostly seen: bankruptcy, failure to pay, restructuring, repudiation/moratorium, obligation acceleration and obligation default.

4. *CDS Spread* - The periodic payments from the protection buyers to protection sellers are called CDS spread. To ensure the absence of arbitrage, the spreads paid to the protection seller should be equal to the expected financial loss in the event of credit default. The CDS spread at issue date is derived by making the present value of the premium payments equal to the present value of the payments at default. After the issue date, the CDS spread is fluctuating according to the market conditions. For example, if the credit risk of a specified reference entity increases, the CDS spread will increase to provide sufficient compensation for the CDS seller undertaking the increased level of credit risk. On the contrary, less spread indicates less credit risk of the specified reference entity. A typical payment term in the contract is "quarterly".

5. *CDS Maturity* - The maturity of a CDS contract ranges from a few months to 30 years.

However, the 5-year CDS contracts have the highest liquidity among all CDS contracts with different maturities.

1.2 Credit Risk Models

Theoretical pricing on credit default swap becomes a key component in credit risk modeling. Credit risk models can be classified into two main categories: structural models and reduced form models. Structural models assume that a company defaults on its debts when the total asset value of the firm falls below a default barrier. Structural models consider reasonable economic variables as inputs e.g. long term debt, total asset value, short-term debt. The parameter calibration of structural models on historical CDS spread is not easy. Reduced form models assume that the default occurs at a random stopping time with stochastic intensity. Reduced form models consider the default as a Poisson process. Compared with structural models, reduced form models can fit in the historical CDS spread data but lack of intuitive economic interpretations.

1.2.1 Structural Models

Structural model is developed by Merton (1974) who uses the Black and Scholes option pricing formulas as well as the company's total assets value to price company equity and risky debt. However, Merton's model simplifies the real world and presents several shortcomings listed below:

First, the event of a default only occurs at the maturity of a debt. In fact, the credit default could happen at any time before or after maturity of a debt. Black and Cox (1976) developed a first passage time structural model which allows the credit default to occur at any time. It assumes that a company defaults on its debts when the total asset value of the firm falls below a default barrier.

Second, the Merton model assumes a constant interest rate. However, the term structure of interest rates follows a stochastic process. Longstaff and Schwartz (1995) apply Vasicek's (1977) interest rate term structure model to price the risky fixed and floating rate debts.

Third, the Merton model assumes a simple debt structure. For example, Merton model assumes zero-coupon bonds. Leland et al. (1994) and Leland and Toft (1996)'s first passage time structural models allow continuous coupons and other possible debt structures.

Finally, Merton's model cannot capture the instantaneous default events. Zhou (2001b) adopts a jump-diffusion process to model the default process which allows immediate default. In addition, Zhou (2001a) calculates the default probability between two companies.

1.2.2 Empirical Results of Structural Models

The empirical results of the structural model turn out to be not satisfying. The estimations of credit spreads and CDS spreads by structural models are not consistent with observed data. Jones et al. (1984) presents the first empirical study on Merton model

(1974) and finds that the bond prices by Merton model (1974) are far below the observed market prices. Eom et al. (2004) implements the structural models of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001) using 182 non-callable bonds of simple capital structure firms between 1986 and 1997. They find that Merton (1974) model underestimates the corporate bond spreads while most of the other structural models overestimate the corporate bond spreads. Ericsson et al. (2006) tests three structure models of Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000) using CDS data from June 1997 to April 2003. They find that the CDS spreads by Leland (1994) model and Fan and Sundaresan (2000) model are too low and the CDS spreads by the Leland and Toft (1996) are too high compared to the historical CDS data. The parameter calibration of structural models to fit historical CDS spread is not straightforward. Huang and Huang (2012) use historical CDS data such as default probability, leverage ratio, default loss and equity risk premium as input to fit the parameters of structural models. They find that credit risk is not a key component in investment- grade bond yields across different maturities. Huang and Zhou (2008) compare five models of Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), Huang and Huang (2012) and Collin-Dufresne and Goldstein (2001). They find that Huang and Huang (2012) and Collin-Dufresne and Goldstein (2001) perform better than Merton model (1974), Black and Cox (1976) and Longstaff and Schwartz (1995) in estimating CDS spreads.

1.2.3 Reduced Form Models

Reduced form models (intensity model) are first proposed by Jarrow and Turnbull (1995) and Duffie and Singleton (1999). The default time is modeled independently from the capital structure of a given firm. The default is driven by unpredictable Poisson events with certain default rate (default intensity). Reduced form models calibrate defaults directly from market prices. Jarrow and Turnbull (1995) derived a close-form solution for a default-risky coupon bond where the default is based on a Poisson process with constant default intensity. Duffie and Singleton (1999) uses a default-adjusted short-rate process in which the sum ofa risk free interest rate and risk premium term is used to represent default-adjusted interest rates. This default-adjusted short-rate process is the discounting factor to account for the default probability, default time and recovery rate. Hull and White (2000) use a default density term structure derived from prices of zero-coupon corporate bonds and treasury bonds instead of default intensity to calculate CDS spreads. Giesecke et al. (2010) propose an exact simulation method for dynamic intensity-based model to analyze correlated defaults risk.

1.2.4 Empirical Results of Reduced Form Models

Longstaff et al. (2005) uses bond market data to predict the CDS spreads and finds that the resulting spreads are higher than market CDS spreads due to liquidity and tax effects. They find that the CDS market is a good source to obtain the credit premium. Blanco et al. (2005) also confirms that the CDS market data directly reflects the credit risk of the issuers. Duffie (1999) implements the Duffie and Singleton (1999) reduced form model to fit the market data of credit premium and CDS spread. The default intensity and the recovery rate jointly determine the CDS spread in the reduced form model. The reduced form model is tractable in fitting the observed CDS spread, but lack of intuitive economic explanations. Duffie (2010) imbedded the following four factors in structural models, the distance to default, the stock return, the market return and the short-term interest rates into reduced form models to predict CDS spread. Duffie (2010) combines the structural model and reduced form model to predict CDS spreads. This could be a new research direction in this area.

1.3 Credit Default Swaps Valuation

We assume that the valuation of CDS is under risk neutral measure. The present value of a CDS contract is equal to the present value of all its future cash flows discounted at risk free rate. For simplicity, we make the following assumptions:

a. The recovery rate is constant (e.g. 40%);

b. The risk-free interest rate is also constant (e.g. 0.00132%);

c. Counter party credit risk and upfront spread payments are not considered in our pricing model;

d. Each CDS has a notional principal of US $1.

The present value of the cash flows from the protection buyer consists of the present value of the periodic payments and the accrual payment that occurs when defaults happen between two payment dates. The present value of the cash flows from the protection seller is equal to the financial loss due to default occuring. A CDS spread is the premium, which makes the present value of the cash flows from the protection buyer equal to the present value of the cash flows from the protection seller. According to Hull et al. (2010) and Yu (2007), we have the formula of a CDS as the following:

$$s = \frac{\sum_{w=1}^{10^6}\Big((1-R)B\big(0,\tau_{ref}(w)\big)1\big(\tau_{ref}(w)\le T,\tau_{sel}(w)>T\big)\Big)}{\sum_{w=1}^{10^6}\Big(\sum_{i=0}^{N}\Big(\Delta t B(0,t_i)1\big(\tau_{ref}(w)>t_i,\tau_{buy}(w)>t_i\big)+\big(\tau_{ref}(w)-t_i\big)B\Big(0,\tau_{ref}(w)\Big)1\big(t_{i+1}>\tau_{ref}(w)>t_i,\tau_{buy}(w)>t_i\big)\Big)\Big)}$$

where $N = \frac{T}{\Delta t}$, R is the recovery rate, τ_{buy} is the first passage time of protection buyer's total asset value reaching the default barrier, τ_{sel} is the first passage time of protection seller's total asset value reaching the default barrier, τ_{ref} is the first passage time of the reference entity's total asset value reaching the default barrier and $B(0, t)$ is the price of a risk-free zero coupon bond maturing at time t with payoff \$1. At default time τ, the protection seller pays the corresponding financial loss of $(1 - R)$ to the protection buyer. Let us denote N as the total number of payments from protection buyer and t as the time period between each payment. The formula can be used to forecast the value of the CDS spreads given that the default time τ_{buy}, τ_{sel}, τ_{ref} are determined by a first passage time model. In the next section, we focus on the calculation of the correlated default times of protection buyer, protection seller and reference entity τ_{buy}, τ_{sel}, τ_{ref}.

In the first passage time models, default occurs when equity prices reach zero. In the Black and Cox (1976) model, default occurs when the value of the assets of a firm reaches a barrier, not necessarily equal to the principal of the debt. Zhou (2001a) found the exact distribution of first passage times of logarithm of the asset value of two firms. Unfortunately, Zhou's analytical formulas cannot be extended to apply to more than two firms. The default events of firms are correlated. The default correlation cannot be neglected in credit risk modeling. In this paper, we propose a new method to simulate the default times for multiple firms (not restricted to two firms) by taking into account the default correlations. Our simulation method can easily handle more than 2 firms' first passage times while Zhou's method fails. The first passage times are used to value the CDS spreads.

2. Model Development

2.1 Joint Density of First Passage Time for Correlated Brownian Motions

Let us start with a two dimensional problem (suppose there are only two firms in a CDS portfolio). Let $X_1(t)$ and $X_2(t)$ denote the logarithm of the total asset values of firm 1 and firm 2. The dynamics of $X_1(t)$ and $X_2(t)$ are given by

$$dX_1(t) = \mu_1 dt + \sigma_1 dW_1(t), X_1(0) = x_1$$

$$dX_2(t) = \mu_2 dt + \sigma_2 dW_2(t), X_2(0) = x_2$$

where $\mu_1, \mu_2 \in \mathbb{R}$ are constant drift terms, $\sigma_1, \sigma_2 > 0$ are constant standard deviation terms, and W_1, W_2 are standard correlated Brownian motions with correlation ρ which reflects the correlation between the movement in the logarithm of the two firms' asset values. Let τ_1, τ_2 be the first passage time that the process $X_1(t)$ and $X_2(t)$ starting from x_1, x_2 reaches the fixed barrier b_1, b_2. If the logarithm of a firm's asset value $X_i(t)$, $(i = 1, 2)$ falls to the threshold level b_i, the firm defaults on its obligations. Let us first introduce the joint density of the first passage time for a couple of correlated Brownian motions.

Proposition 1.1: Let $f^{\mu_1,\mu_2}(s,t)$ denote the joint density of the first passage time for the process $X_1(t)$, $X_2(t)$ in the presence of drift μ_1, μ_2, where $s = \tau_1$ and $t = \tau_2$. When $sgn\left(\frac{x_1-b_1}{\mu_1}\right) < 0$ and $\left(\frac{x_2-b_2}{\mu_2}\right) < 0$, we obtain the joint density $f^{\mu_1,\mu_2}(s,t)$ of (τ_1, τ_2) for correlated Brownian motion $X_1(t)$, $X_2(t)$ as follows:

$$f^{\mu_1,\mu_2}(s,t)$$

$$= \begin{cases} \sqrt{\frac{\pi}{2(t-s)^3}} \frac{\sin(\alpha)}{\alpha^2 s} \exp\left(-\frac{\tilde{\gamma}_1^2 s + \tilde{\gamma}_2^2 t}{2} - r_0(\tilde{\gamma}_1 \cos(\theta_0) + \tilde{\gamma}_2 \sin(\theta_0)) - \frac{r_0^2}{2s}\right) \\ \times \int_0^\infty \sum_{n=1}^\infty n \sin\left(\frac{n\pi(\alpha-\theta_0)}{\alpha}\right) \exp(\tilde{\gamma}_1 \mathrm{r}\cos(\alpha) - \frac{r^2(t-s\cos(\alpha)^2)}{2s(t-s)}) \times I_{\frac{n\pi}{\alpha}}\left(\frac{rr_0}{s}\right) dr, & 0 < s < t \\ \sqrt{\frac{\pi}{2(s-t)^3}} \frac{\sin(\alpha)}{\alpha^2 t} \exp\left(-\frac{(s-t)(\tilde{\gamma}_1 \sin(\alpha) - \tilde{\gamma}_2 \cos(\alpha))^2}{2} - r_0(\tilde{\gamma}_1 \cos(\theta_0) + \tilde{\gamma}_2 \sin(\theta_0)) - \frac{\tilde{\gamma}_1^2 + \tilde{\gamma}_2^2}{2} t - \frac{r_0^2}{2t}\right) \\ \times \int_0^\infty \sum_{n=1}^\infty n \sin\left(\frac{n\pi\theta_0}{\alpha}\right) \times \exp\left(-\frac{r^2(s-t\cos(\alpha)^2)}{2t(s-t)} - r(\tilde{\gamma}_1 \sin^2(\alpha) - \tilde{\gamma}_2 \sin(\alpha)\cos(\alpha) - \tilde{\gamma}_1)\right) \times I_{\frac{n\pi}{\alpha}}\left(\frac{rr_0}{t}\right), & 0 < t < s \end{cases}$$

where $I_k(x)$ is the modified Bessel function I with order k and $\tilde{\rho} = \left(sgn\left(\frac{x_1-b_1}{x_2-b_2}\right)\right)\rho$,

then

$$\alpha = \begin{cases} \pi + \tan^{-1}\left(\frac{\sigma_1\sqrt{1-\tilde{\rho}^2}}{\sigma_2 - \tilde{\rho}\sigma_1}\right), & \sigma_1\tilde{\rho}|x_2-b_2| > |x_1-b_1|\sigma_2 \\ \frac{\pi}{2}, & \sigma_1\tilde{\rho}|x_2-b_2| = |x_1-b_1|\sigma_2 \\ \tan^{-1}\left(\frac{\sigma_1\sqrt{1-\tilde{\rho}^2}}{\sigma_2 - \tilde{\rho}\sigma_1}\right), & \sigma_1\tilde{\rho}|x_2-b_2| < |x_1-b_1|\sigma_2 \end{cases}$$

$$\theta_0 = \begin{cases} \pi + \tan^{-1}(-\frac{\sqrt{1-\tilde{\rho}^2}}{\tilde{\rho}}), & \tilde{\rho} > 0 \\ \frac{\pi}{2} & ,\tilde{\rho} = 0 \\ \tan^{-1}(-\frac{\sqrt{1-\tilde{\rho}^2}}{\tilde{\rho}}) & ,\tilde{\rho} < 0 \end{cases}$$

$$r_0 = \frac{1}{\sigma_1\sigma_2}\sqrt{\frac{(x_2-b_2)^2\sigma_1^2 + (x_1-b_1)^2\sigma_2^2 - 2|(x_1-b_1)(x_2-b_2)|\tilde{\rho}\sigma_1\sigma_2}{1-\tilde{\rho}^2}}$$

$$\gamma_1 = \frac{\sigma_2\mu_1 - \sigma_1\mu_2\rho}{\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

$$\gamma_2 = \frac{\mu_2}{\sigma_2}$$

$$\tilde{\gamma}_1 = sgn(x_1 - b_1)\gamma_1$$

$$\tilde{\gamma}_2 = sgn(x_2 - b_2)\gamma_2$$

Metzler (2010) only derived $f^{\mu_1,\mu_2}(s,t)$ when $0 < s < t$. We continue to derive $f^{\mu_1,\mu_2}(s,t)$ when $0 < t < s$. The general version of joint density for the first passage time (τ_1, τ_2) is listed in equation 1.

2.2 First Passage Time Algorithm for Correlated Brownian Motions

When $\mu_1 = \mu_2 = 0$, we denote the joint density for the first passage time to be $f^{0,0}(s,t)$. In this section, the algorithm to simulate the first passage time according to the joint density for the first passage time (τ_1, τ_2) when $\mu_1 = \mu_2 = 0$ is presented. If $\mu_1 = \mu_2 = 0$, then there exists χ_1^2, χ_2^2, two Chi-square random variables such that, with probability 1,

$$(\tau_1, \tau_2) = \left(\frac{(x_1 - b_1)^2}{\sigma_1^2 \chi_1^2}, \frac{(x_2 - b_2)^2}{\sigma_2^2 \chi_2^2}\right).$$

In order to get (τ_1, τ_2) , simulating two Chi-square random variables χ_1^2, χ_2^2 is the key,

$$(\chi_1^2, \chi_2^2) = \left(\frac{(x_1 - b_1)^2}{\sigma_1^2 \tau_1}, \frac{(x_2 - b_2)^2}{\sigma_2^2 \tau_2}\right).$$

Let $U_1 \sim Unif(0,1), U_2 \sim Unif(0,1)$

$$(U_1, U_2) = \left(F_{\chi^2}\left(\frac{(x_1 - b_1)^2}{\sigma_1^2 \tau_1}\right), F_{\chi^2}\left(\frac{(x_2 - b_2)^2}{\sigma_2^2 \tau_2}\right)\right).$$

Let $Z \sim N(0,1)$ and we observe that for any $X \geq 0$,

$$F_{\chi^2}(X) = P(Z^2 \leq X) = P\left(-\sqrt{X} \leq Z \leq \sqrt{X}\right) = 2\Phi\left(\sqrt{X}\right) - 1\,.$$

Then the calculation of (U_1, U_2) can be given by using the joint density of first passage time (τ_1, τ_2):

$$
\begin{aligned}
Corr(U_1, U_2) &= Corr\left(F_{\chi^2}\left(\frac{(x_1-b_1)^2}{\sigma_1^2}\right), F_{\chi^2}\left(\frac{(x_2-b_2)^2}{\sigma_2^2}\right)\right) \\
&= Corr\left(2\Phi\left(\frac{|x_1-b_1|}{\sigma_1\sqrt{\tau_1}}\right)-1, 2\Phi\left(\frac{|x_2-b_2|}{\sigma_2\sqrt{\tau_2}}\right)-1\right) \\
&= \frac{E\left[(2\Phi\left(\frac{|x_1-b_1|}{\sigma_1\sqrt{\tau_1}}\right)-1)(2\Phi\left(\frac{|x_2-b_2|}{\sigma_2\sqrt{\tau_2}}\right)-1)\right]-E(U_1)E(U_2)}{\sqrt{Var(U_1)VaR(U_2)}} \\
&= 12\left[\int_0^{\infty}\int_0^{\infty}\left(2\phi\left(\frac{|x_1-b_1|}{\sigma_1\sqrt{\tau_1}}\right)-1\right)\left(2\phi\left(\frac{|x_2-b_2|}{\sigma_2\sqrt{\tau_2}}\right)-1\right)\times f^{0,0}(s,t)dsdt \right.\\
&\quad \left. -\frac{1}{4}\right].
\end{aligned}
\tag{2}
$$

Let $(Z_1, Z_2) \sim N(0,1)$ with correlation denoted as $\text{Corr}(Z_1, Z_2)$,

$$Corr(Z_1, Z_2) = 2\sin(\frac{\pi}{6} Corr(U_1, U_2)) \,. \tag{3}$$

The simulation of (χ_1^2, χ_2^2) is given by:

$$\chi_1^2 = F_{\chi^2}^{-1}(U_1) = F_{\chi^2}^{-1}(\Phi(Z_1)) = \Phi^{-1}\left(\frac{1+\Phi(Z_1)}{2}\right)^2 ,$$

$$\chi_2^2 = F_{\chi^2}^{-1}(U_2) = F_{\chi^2}^{-1}(\Phi(Z_2)) = \Phi^{-1}(\frac{1+\Phi(Z_2)}{2})^2 \,. \tag{4}$$

Finally, we have

$$(\tau_1, \tau_2) = (\frac{(x_1-b_1)^2}{\sigma_1^2\chi_1^2}, \frac{(x_2-b_2)^2}{\sigma_2^2\chi_2^2}) \,. \tag{5}$$

Algorithm 1 is summarized in the following for simulating the first passage time of two correlated Brownian motions given $\mathrm{Corr}(W_1(t), W_2(t))$

Algorithm 1 Correlated Brownian Motions
1. Calculate $\mathrm{Corr}(U_1, U_2)$ by equation 2;
2. Simulate $(Z_1, Z_2) \sim N(0,1)$ with correlation $\mathrm{Corr}(Z_1, Z_2)$ by equation 3;
3. Simulate (χ_1^2, χ_2^2) by equation 4;
4. Simulate (τ_1, τ_2) by equation 5.

2.3 First Passage Time Algorithm for Correlated Arithmetic Brownian Motions

Now we move to the general case when the drifts μ_1, μ_2 are non-zero. Let us denote IG to be inverse Gaussian distribution. When $sgn\left(\frac{x_1-b_1}{\mu_1}\right) < 0$ and $sgn\left(\frac{x_2-b_2}{\mu_2}\right) < 0$, we observe that $\tau_1 \sim IG(\lambda_1^{(1)}, \lambda_2^{(1)})$ and $\tau_2 \sim IG(\lambda_1^{(2)}, \lambda_2^{(2)})$ where

$$\lambda_1^{(1)} = \frac{x_1 - b_1}{\mu_1}; \quad \lambda_2^{(1)} = \frac{(x_1 - b_1)^2}{\sigma_1^2}; \lambda_1^{(2)} = \frac{x_2 - b_2}{\mu_2}; \quad \lambda_2^{(2)} = \frac{(x_2 - b_2)^2}{\sigma_2^2}.$$

When $sgn\left(\frac{x_1-b_1}{\mu_1}\right) < 0$ and $sgn\left(\frac{x_2-b_2}{\mu_2}\right) < 0$, the following relation holds:

$$\left(\frac{\lambda_2^{(1)}\left(\tau_1 - \lambda_1^{(1)}\right)^2}{\left(\lambda_1^{(1)}\right)^2 \tau_1}, \frac{\lambda_2^{(2)}\left(\tau_2 - \lambda_1^{(2)}\right)^2}{\left(\lambda_1^{(2)}\right)^2 \tau_2}\right) \sim (\chi_1^2, \chi_2^2),$$

where χ_1^2, χ_2^2 are two Chi-square random variables. In order to simulate (τ_1, τ_2), we start with simulating the correlated Chi-square random variables (χ_1^2, χ_2^2). Let

$U_1 \sim Unif(0,1), U_2 \sim Unif(0,1)$,

$$(U_1,U_2)=\left(F_{\chi^2}\left(\frac{\lambda_2^{(1)}\left(\tau_1-\lambda_1^{(1)}\right)^2}{\left(\lambda_1^{(1)}\right)^2\tau_1}\right),F_{\chi^2}\left(\frac{\lambda_2^{(2)}\left(\tau_2-\lambda_1^{(2)}\right)^2}{\left(\lambda_1^{(2)}\right)^2\tau_2}\right)\right).$$

Let $Z\sim N(0,1)$. Then for any $X\geq 0$,

$$F_{\chi^2}(X)=P(Z^2\leq X)=P\left(-\sqrt{X}\leq Z\leq \sqrt{X}\right)=2\Phi\left(\sqrt{X}\right)-1\,.$$

Then the calculation of (U_1,U_2) can be given by using the joint density of first passage time (τ_1,τ_2).

$$Corr(U_1,U_2)=Corr\left(F_{\chi^2}\left(\frac{\lambda_2^{(1)}(\tau_1-\lambda_1^{(1)})^2}{(\lambda_1^{(1)})^2\tau_1}\right),F_{\chi^2}\left(\frac{\lambda_2^{(2)}(\tau_2-\lambda_1^{(2)})^2}{(\lambda_1^{(2)})^2\tau_2}\right)\right)$$

$$=Corr\left(2\Phi\left(\frac{\sqrt{\lambda_2^{(1)}}\left|\tau_1-\lambda_1^{(1)}\right|}{\left|\lambda_1^{(1)}\right|\sqrt{\tau_1}}\right)-1,2\Phi\left(\frac{\sqrt{\lambda_2^{(2)}}\left|\tau_2-\lambda_1^{(2)}\right|}{\left|\lambda_1^{(2)}\right|\sqrt{\tau_2}}\right)-1\right)$$

$$=\frac{E\left[(2\Phi\left(\frac{\sqrt{\lambda_2^{(1)}}\left|\tau_1-\lambda_1^{(1)}\right|}{\left|\lambda_1^{(1)}\right|\sqrt{\tau_1}}\right)-1)(2\Phi\left(\frac{\sqrt{\lambda_2^{(2)}}\left|\tau_2-\lambda_1^{(2)}\right|}{\left|\lambda_1^{(2)}\right|\sqrt{\tau_2}}\right)-1)\right]-E(U_3)E(U_4)}{\sqrt{Var(U_3)VaR(U_4)}}$$

$$=12\left[\int_0^\infty\int_0^\infty\left(2\Phi\left(\frac{\sqrt{\lambda_2^{(1)}}\left|\tau_1-\lambda_1^{(1)}\right|}{\left|\lambda_1^{(1)}\right|\sqrt{\tau_1}}\right)-1\right)\left(2\Phi\left(\frac{\sqrt{\lambda_2^{(2)}}\left|\tau_2-\lambda_1^{(2)}\right|}{\left|\lambda_1^{(2)}\right|\sqrt{\tau_2}}\right)-1\right)\right.$$

$$\left.\times f^{\mu_1,\mu_2}(s,t)dsdt-\frac{1}{4}\right]$$

(6)

Let $(Z_1, Z_2) \sim N(0,1)$ with correlation $\mathrm{Corr}(Z_1, Z_2)$

$$Corr(Z_1, Z_2) = 2\sin\left(\frac{\pi}{6} Corr(U_1, U_2)\right). \tag{7}$$

The simulation of (χ_1^2, χ_2^2) is given by:

$$\chi_1^2 = \Phi^{-1}\left(\frac{1+\Phi(Z_1)}{2}\right)^2; \chi_2^2 = \Phi^{-1}\left(\frac{1+\Phi(Z_2)}{2}\right)^2. \tag{8}$$

By using the similar method in Michael et al.'s method (1976), we can generate the first passage time approximately starting from the joint density distribution of (χ_1^2, χ_2^2). The simulation of the first exit time for two correlated arithmetic Brownian motions is given in the following theorem.

Theorem 1.3: Let

$$(\chi_1^2, \chi_2^2) \sim \left(\frac{\lambda_2^{(1)}\left(\tau_1 - \lambda_1^{(1)}\right)^2}{\left(\lambda_1^{(1)}\right)^2 \tau_1}, \frac{\lambda_2^{(2)}\left(\tau_2 - \lambda_1^{(2)}\right)^2}{\left(\lambda_1^{(2)}\right)^2 \tau_2}\right),$$

For $i = 1,2$, set

$$X_{i1} = \lambda_1^i + \frac{(\lambda_1^{(i)})^2 \chi_i^2}{2\lambda_2^{(i)}} - \frac{\left|\lambda_1^{(i)}\right|}{2\lambda_2^{(i)}} \sqrt{4\lambda_1^{(i)}\lambda_2^{(i)}\chi_i^2 + (\lambda_1^{(i)})^2(\chi_i^2)^2},$$

$$X_{i2} = \frac{(\lambda_1^{(i)})^2}{X_{i1}}.$$

For $u, v \in \{1,2\}$, define

$$P_{uv} = \left(1 + \sum_{(i,j)\in\{1,2\}^2\setminus\{(u,v)\}} \left| \frac{\left(X_{1u}^2 - \left(\lambda_1^{(1)}\right)^2\right)\left(X_{2v}^2 - \left(\lambda_1^{(2)}\right)^2\right)\left(X_{1i}X_{2j}\right)^2}{\left(X_{1i}^2 - \left(\lambda_1^{(1)}\right)^2\right)\left(X_{2j}^2 - \left(\lambda_1^{(2)}\right)^2\right)\left(X_{1u}X_{2v}\right)^2} \right| \frac{f^{\mu_1,\mu_2}\left(X_{1i}, X_{2j}\right)}{f^{\mu_1,\mu_2}\left(X_{1u}, X_{2v}\right)} \right)^{-1}, \tag{9}$$

where f^{μ_1,μ_2} is the joint density of (τ_1, τ_2). Construct a partition of interval [0, 1]:

$$I_{11} = [0, P_{11}),$$

$$I_{12} = [P_{11}, P_{11} + P_{12}),$$

$$I_{21} = [P_{11} + P_{12}, P_{11} + P_{12} + P_{21}),$$

$$I_{22} = [P_{11} + P_{12} + P_{21}, 1].$$

Then we have

$$(\tau_1, \tau_2) \sim \sum_{(i,j)\in\{1,2\}} (X_{1i}, X_{2j})\, \chi_U \in I_{i,j}(U),$$

where $U \sim Unif(0,1)$ is an uniform random variable independent of χ_1^2, χ_2^2 and $\chi_U \in I_{i,j}(U)$ is an indicator function. If U is an element in a subset $I_{i,j}$, the indicator function returns1. Otherwise, the indicator function returns 0.

Algorithm 2 is summarized in the following for the simulation of the first passage time of two correlated arithmetic Brownian motions given Corr$(W_1(t), W_2(t))$:

Algorithm 2 Correlated Arithmetic Brownian Motions
1. Calculate Corr(U_1, U_2) by equation 6;
2. Simulate $(Z_1, Z_2) \sim N(0,1)$ with correlation Corr(Z_1, Z_2) by equation 7;
3. Simulate (χ_1^2, χ_2^2) by equation 8;
4. Calculate $X_{11}, X_{21}, X_{12}, X_{22}, P_{11}, P_{21}, P_{12}, P_{22}$ by equation 9;

5. Let $U \sim Unif(0,1)$;
   If $U \in [0, P_{11})$
   $\tau_1 = X_{11}$
   $\tau_2 = X_{21}$
   elseif $U \in [P_{11}, P_{11} + P_{12})$
   $\tau_1 = X_{11}$;
   $\tau_2 = X_{22}$;
   elseif $U \in [P_{11} + P_{12}, P_{11} + P_{12} + P_{21})$
   $\tau_1 = X_{12}$;
   $\tau_2 = X_{21}$;
   elseif $U \in [P_{11} + P_{12} + P_{21}, 1]$
   $\tau_1 = X_{12}$;
   $\tau_2 = X_{22}$;
   End

2.4 Extension to High Dimensional Correlated Brownian Motions

The number of firms determines the number of dimensions of the problems. In section 2.2, and 2.3, only two dimensional (two companies) algorithms are presented. The algorithms to simulate the first passage time of multiple correlated Brownian motions without drift are proposed in this part. When drift terms are zero, we can easily expand Algorithm 1 to high dimensions. However, for high dimensional arithmetic Brownian motions, the simulation of (τ_1 , τ_2 ,…, τ_N) becomes intractable. By using the two dimensional joint density, we can only simulate a couple of first passage time (τ_1 , τ_2 ,…, τ_N). Algorithm 3 is summarized in the following to simulate the first passage time of multiple ($N > 2$) correlated Brownian motions when drift terms are zero:

Algorithm 3 High Dimensional Correlated Brownian Motions
1. Calculate Corr(U_1, U_2), Corr(U_1, U_3) , Corr(U_2, U_3) ... by equation 2;
2. Simulate $(Z_1, Z_2, Z_3 \ldots) \sim N(0,1)$ with correlation Corr(Z_1, Z_2), Corr(Z_1, Z_3), Corr(Z_2, Z_3) … by equation 3;
3. Simulate $(\chi_1^2, \chi_2^2, \chi_3^2, \ldots)$ by equation 4;
4. Simulate $(\tau_1, \tau_2, \tau_{3,} \ldots)$ by equation 5.

3. Data and Methodology

CDS data has been obtained from Bloomberg terminal. Since the 5-year CDS contracts have the highest liquidity among all CDS contracts across different maturities, we use the 5-year CDS data in this analysis. This led to an initial dataset of 109 observations from Bloomberg as of June 30th, 2013. We removed 28 observations that were due to lack of financial data and removed another 7 observations that were due to duplicate records/reporting in Bloomberg. In addition, we excluded 14 observations that were not actively traded. We finalized a sample of 60 CDS with their prices and the corresponding credit spreads from the underlying companies on June 30th, 2013. We also downloaded the following data from Bloomberg for these 60 companies: total asset value, short-term liability, long-term liability, stock price, credit rating, and implied equity volatility. The information on the identity of buyer and seller of each CDS contract is not public. According to CME group's report, Bank of America, JPMorgan Chase & Co, Barclay PLC, Citigroup, Credit Suisse AG, Deutsche Bank AG, The Goldman Sachs Group, HSBC Holding and Morgan Stanley are the major dealers in CDS market. In this paper, we assume these major dealers are the buyers and sellers in CDS contracts.

The model assumes that the logarithm of the current total asset values of firms follows a Brownian motion,

$$dX_i(t) = \mu_i dt + \sigma_i dW_i(t), \qquad X_i(0) = x_i.$$

where i represents firm number. To apply the model, we need to calibrate the parameters of the default barrier and current total asset value. The substantial decrease of a firm's asset value is the main reason of the default of the firm. According to Black and Cox (1976), the barrier is equal to $e^{a_i t}K_i$ where $a = \mu_i$ (assume the debt value of a firm and the firm's total asset values have the same growth rate). The drift terms of the logarithm of the total asset values and the barriers cancel each other. It becomes a problems of calculating the first passage time for the Brownian motions without drift to reach the value of K_i. Black and Cox (1976) defined the threshold value K_i as the minimum asset value of the firm required by the debt covenants. If the firm's asset value falls to a certain threshold value K_i, the bondholders are entitled to a "deficiency claim" which can force the firm into bankruptcy. Zhou (2001) and Leland (2006) defined the barrier K_i as the logarithm of the sum of the short-term debt principal and one half of the long-term debt principal of a firm. We assume that the volatilities of the total asset values are equal to its stock's volatilities, which are easily observable. The correlations of the companies' asset values are equal to the companies' stock correlations. In the following section, the CDS spread is measured in basis points according to Longstaff et al. (2005).

4. Empirical Results

Table 1 presents the descriptive statistics for the variables used in our analysis. The CDS spreads calculated by new proposed model (including effect of the counterparty risk) have significantly lower CDS premiums with t-statistic equal to 6.1338 and p-value equal to 0. Since we use CDS contracts written on financial services firms, the firms in the sample are similar to each other. However, the conditional default probability which measures the counterparty risk directly has a larger variation. Another interesting observation from Table 1 is that the majority of the underlying bonds for these CDS instruments have high credit ratings. In fact, there are only three CDS contracts that are written on below-investment grade bonds in our samples. To some extent, this is due to sample selection, and/or the unavailability of debt in the market for those below investment-grade firms. Results in table 2 show that the correlation between historical spreads and calculated spreads are still high, namely 0.7815, statistically significant at 1%.

In Table 3, we analyze the determinants of historical CDS spreads. The debt ratio, own default probability and log firm size are statistically significant while the implied Black-Scholes volatility obtained from at-the-money stock options, cash ratio and credit rating are statistically not significant. As expected, the historical CDS spreads have strong positive linear relationship with the reference entity's default probability. Strong and positive linear relationship between debt ratios and the historical CDS spreads can be easily explained by the fact that an increase in the debt of the underlying company leads to a higher likelihood of failure of that company. Consequently, an increase in likelihood of failure of the company coincides with a higher CDS spread.

In Table 4, we explore the determinants of the deviation from historical CDS spreads. The debt ratio, joint default probability, conditional probability and the log of firm size are highly statistically significant in explaining the deviation from the historical CDS spreads. The coefficients of debt ratio variable (e.g. 153.549) reflect high influence on deviation from the historical CDS spreads. An increase of the debt ratio represents a higher likelihood of the reference entity's default, which directly leads to a higher CDS premium. We notice that the coefficients of joint default probability variables and conditional probability variables are extremely high (e.g. -573.6 & -801.14) when compared with other variables. We infer that the counterparty risk (i.e. conditional default probability of the seller default given the reference entity does not default) is one of the major determinants in explaining the deviation from historical CDS spread.

5. Conclusion

This paper presents an extended structured model on CDS pricing by taking into account effect of counterparty risk. The expanded structural model captures the significant changes in total asset values of counterparties while the reduced-form model fails to do it. We find that the CDS spread is significantly affected by counterparty risk. Based on empirical results, the counterparty risk is not priced in the credit derivative markets, and CDS issued by most financial firms are overpriced. We analyze the difference between the historical spreads and the model spreads and find that the discrepancy lies in the imperfect proxies of counterparty risk used in CDS pricing.

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Table I
Descriptive Statistics

Historical CDS spread is the CDS spreads which are observable in the market. Calculated spread is the CDS spreads calculated from new proposed model. Historical spread and calculated spread are all quoted in basis points. In the new proposed model, we assume JPMorgan Chase & Co is the seller and Bank of America is the buyer in 58 CDS contracts. Log of firm sizes in the table are logarithm of the firm total asset value. The credit rating represents the long-term debt rating of the firm by Standard and Poor's. We quantify the credit ratings numerically from 1 up to 15 where 15 represents the highest AAA rating and each rating levels descends by 1 point from there. Debt ratio represents the ratio of total debt (the sum of current liabilities and long-term liabilities) and total assets (the sum of current assets, fixed assets, and other assets). Cash ratio represents the ratio of cash and cash equivalents of a company to its current liabilities. We assume the total asset as the most significant measures of the firm size. Joint default probability is the probability of both the reference entity and the seller default before maturity date of CDS contract. Conditional default probability is the probability of the seller defaults while the reference entity does not default before maturity date of CDS contract. Own default probability represents the default probability of the reference entity itself. Implied volatility is derived from the Black-Scholes Model on the company's equity options. The barrier is equal to logarithm of the sum of short-term debt principal plus one-half of long-term debt principal.

	Mean	**Median**	**Std**	**Min**	**Max**
Historical Spread (bp)	132.542	106.889	82.5925	31.585	862.8364
Calculated Spread (bp)	42.9623	95.041	43.3963	2.8059	206.37
Credit Rating	9.8833	10.00	2.07562	3.00	15.00
Cash Ratio	0.74235	0.2403	1.5223	0.001	9.3076
Log of Firm Size	24.3371	23.8513	2.0367	21.456	28.522
Debt Ratio	0.7176	0.7529	0.17332	0.4041	0.96635
Joint Default Pro.	0.09135	0.0781	0.06746	0.0002	0.2954
Conditional Default Pro.	0.268877	0.2791	0.05181	0.0871	0.3394
Own Default Pro.	0.15509	0.12903	0.14266	0.0007	0.6891
Implied Volatility	0.24353	0.2234	0.0911	0.1105	0.63266
Barrier	23.5301	22.9114	2.1171	20.258	27.9403

Table II
Correlations

Historical CDS spread is the CDS spreads which are observable in the market. Calculated spread is the CDS spreads calculated from new proposed model. Historical spread and calculated spread are all quoted in basis points. In the new proposed model, we assume JPMorgan Chase & Co is the seller and Bank of America is the buyer in 58 CDS contracts. Pearson p-values are presented in parentheses. *,** and *** indicate statistical significance at 10%, 5% and 1% levels, respectively.

	Historical Spread	Calculated Spread
Historical Spread	1	0.781497***
		(<0.0001)
Joint Default Pro.	0.42953487***	0.67225335***
	(0.00074945)	(<0.0001)
Conditional Default Pro.	-0.4060938***	-0.5580594***
	(0.0001534)	(<0.0001)
Own Default Prob.	0.47927843***	0.757514***
	(<0.0001)	(<0.0001)
Debt Ratio	0.144177018	0.080822237
	(0.280084088)	(0.546349474)
Credit Rating	-0.377361***	-0.4425296***
	(0.00345277)	(0.00049228)
Cash Ratio	0.02083775	0.00386227
	(0.87659884)	(0.97704134)

Table III
Determinants of the Historical CDS Spread

Dependent variable is the historical CDS spread. Log of firm sizes in the table are logarithm of the firm total asset value. The credit rating represents the long-term debt rating of the firm by Standard and Poor's. We quantify the credit ratings numerically from 1 up to 15 where 15 represents the highest AAA rating and each rating levels descends by 1 point from there. Debt ratio represents the ratio of total debt (the sum of current liabilities and long-term liabilities) and total assets (the sum of current assets, fixed assets, and other assets). Cash ratio represents the ratio of cash and cash equivalents of a company to its current liabilities. We assume the total asset as the most significant measures of the firm size. Joint default probability is the probability of both the reference entity and the seller default before maturity date of CDS contract. Conditional default probability is the probability of the seller defaults while the reference entity does not default before maturity date of CDS contract. Own default probability represents the default probability of the reference entity itself. Implied volatility is derived from the Black-Scholes Model on the company's equity options. The barrier is equal to logarithm of the sum of short-term debt principal plus one-half of long-term debt principal. Estimations are White-corrected for heteroskedasticity and t-statistics are represented in parentheses. *,** and *** indicate statistical significance at 10%, 5% and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	305.996***	624.6225***	374.772***	310.98***	333.131***	474.971***
	(3.100967)	(4.570202)	(3.45573)	(2.83567)	(3.37107)	(4.0788)
Implied Volatility	344.0316**					169.0704
	(3.074879)					(1.32534)
Debt Ratio		298.9668***				188.441**
		(3.191489)				(2.4257)
Cash Ratio			0.89452			-8.5998
			(0.12321)			(-1.655549)
Credit Rating				-14.0212		-1.159041
				(-1.59321)		(-0.15255)
Own Default Pro.					278.025***	179.33966**
					(3.2219)	(2.6289)
Log firm size	-10.7563**	-29.2553***	-10.1694**	-1.8095	-10.2637**	-21.9675***
	(-2.489896)	(-3.893942)	(-2.2782)	(-0.279659)	(-2.6213)	(-3.023076)
N	58	58	58	58	58	58
R-square	0.204	0.25078	0.05618	0.1436	0.2865	0.4049
F-value	7.0584***	9.2051***	1.637	4.612**	11.043***	5.784661***

Table IV
Determinants of the Deviation from the Historical CDS Spread

Dependent variable is the difference between our calculated spread and the historical CDS spread. Log of firm sizes in the table are logarithm of the firm total asset value. The credit rating represents the long-term debt rating of the firm by Standard and Poor's. We quantify the credit ratings numerically from 1 up to 15 where 15 represents the highest AAA rating and each rating levels descends by 1 point from there. Debt ratio represents the ratio of total debt (the sum of current liabilities and long-term liabilities) and total assets (the sum of current assets, fixed assets, and other assets). Cash ratio represents the ratio of cash and cash equivalents of a company to its current liabilities. We assume the total asset as the most significant measures of the firm size. Joint default probability is the probability of both the reference entity and the seller default before maturity date of CDS contract. Conditional default probability is the probability of the seller defaults given the reference entity does not default before maturity date of CDS contract. Own default probability represents the default probability of the reference entity itself. Implied volatility is derived from the Black-Scholes Model on the company's equity options. The barrier is equal to logarithm of the sum of short-term debt principal plus one-half of long-term debt principal. Estimations are White-corrected for heteroskedasticity and t-statistics are represented in parentheses. *,** and *** indicate statistical significance at 10%, 5% and 1% levels, respectively.

	(1)	(2)	(3)	(4)
Intercept	324.758***	327.734***	347.844***	655.03029***
	(4.0358)	(4.00579)	(3.797)	(3.6051)
Debt Ratio	163.9749***	168.2809***	149.8822**	153.54979***
	(2.8293)	(2.878819)	(2.4589)	(2.715175)
Joint Default Pro.		-18.37055		-573.598*
				(-1.85536)
Conditional Default Pro.			-77.932	-801.148*
			(-0.52679)	(-1.97725)
Log of Firm Size	-14.669***	-14.8497***	-14.343***	-16.94472***
	(-3.4104)	(-3.4469)	(-3.4047)	(-3.8569)
N	**58**	**58**	**58**	**58**
R-square	0.1498	0.1502	0.1538	0.202469
F-value	4.848**	3.18355**	3.2716**	3.363**